Single Pure - Trigonometry

Patrons are kindly reminded that

$$\sin^2\theta + \cos^2\theta \equiv 1 \qquad \text{and} \qquad \tan\theta \equiv \frac{\sin\theta}{\cos\theta}.$$

They are also kindly reminded that they should know the following:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

They are also reminded to give answers to three significant figures if answers are not exact. Also $\sin^2 x$ means $(\sin x)^2$.

1. What is the exact value of the following (without a calculator)
   (a) $\sin 300$. $\boxed{-\frac{\sqrt{3}}{2}}$
   (b) $\cos(-420)$. $\boxed{\frac{1}{2}}$
   (c) $\tan 300$. $\boxed{-\sqrt{3}}$
2. Sketch
   (a) $y = 2\sin(3x)$ for $0° < x < 180°$.
   (b) $2y = \cos(\frac{x}{2})$ for $-360° < x < 360°$.
3. Convert:
   (a) $\frac{\pi}{10}$ radians to degrees. $\boxed{18}$
   (b) $20°$ to radians. $\boxed{\frac{\pi}{9}}$
   (c) 1.2 radians to degrees. $\boxed{68.8}$
   (d) $540°$ to radians. $\boxed{3\pi}$
4. Solve the following:
   (a) $2\cos x + 1 = 0$ in range $0° < x < 360°$.
   (b) $\tan\theta = \frac{1}{\sqrt{3}}$ in range $0° < x < 360°$.
   (c) $7\sin x = 2\cos x$ in range $0° < x < 360°$. $\boxed{16.0, 195.9}$
   (d) $2\sin x = 1$ in range $0° < x < 360°$. $\boxed{30, 150}$
   (e) $-\sin\theta = 0.6$ in range $-360° < x < 360°$.
   (f) $\tan^2\theta = 2$ in range $-180° < x < 180°$. $\boxed{x = 120, 240}$
5. Solve the following equations in the required range.
   (a) $4\sin^2\theta = 3$ in range $-360° < \theta < 360°$. $\boxed{\theta = \pm 60, \pm 120, \pm 240, \pm 300}$
   (b) $1 + \sin x = 2\cos^2 x$ in range $0° < x < 360°$. $\boxed{x = 30, 150, 270}$
   (c) $5 + 7\sin x = 2\cos^2 x$ in range $-180° < x < 360°$. $\boxed{x = -150, -30, 210, 330}$
   (d) $\cos^2 x + 1 = \sin^2 x + \cos x + 3$ in range $0° < x < 720°$. $\boxed{x = 180, 540}$

(e) $2\tan^2\theta + \tan\theta = 12 + \tan^2\theta$ in range $-360° < \theta < 0°$. $\theta = -288, -256, -108, -76.0$

(f) $2\cos^2\theta + 14 = 8\cos\theta - \sin^2\theta$ in range $-2\pi < \theta < 2\pi$. No solutions

(g) $2\sin x\cos x = \cos x$. (Do *not* divide through by $\cos x$!!!)

6. Solve the following conversions from one trig function to another.

(a) If $\sin\theta = \frac{3}{4}$, find $\cos\theta$. $\pm\frac{\sqrt{7}}{4}$

(b) If $\cos\theta = \frac{5}{6}$, find $\sin\theta$. $\pm\frac{\sqrt{11}}{6}$

(c) If $\cos\theta = -\frac{1}{3}$, find $\tan\theta$. $\pm 2\sqrt{2}$

(d) If $\sin\theta = -\frac{\sqrt{3}}{2}$, find $\cos\theta$. $\pm\frac{1}{2}$

(e) If $\cos\theta = 1$, find $\tan\theta$. 0

(f) If $\tan\theta = \frac{6}{5}$, find $\sin\theta$. $\pm\frac{6}{\sqrt{61}}$

(g) If $\sin\theta = 0.25$, find $\cos\theta$ given that θ is acute. $\frac{\sqrt{15}}{4}$

(h) If $\sin\theta = 0.25$, find $\cos\theta$ given that θ is obtuse. $-\frac{\sqrt{15}}{4}$

(i) If $\sin\theta = \frac{1}{2}$, find $\tan\theta$ given that θ is obtuse. $-\frac{\sqrt{3}}{3}$

7. Solve the following equations.

(a) $\sin 2x = \frac{1}{2}$ for $0° < x < 360°$. $x = 15, 75, 195, 255$

(b) $2\sin\left(\frac{x}{2}\right) = \sqrt{3}$ in range $0 < x < 1440$. $x = 120, 240, 840, 960$

(c) $\sin(2x - 30) = \frac{1}{\sqrt{2}}$ for $-180° < x < 180°$. $x = -142.5, -97, 5, 37.5, 82.5$

(d) $\tan(3x - 10) = 0$ for $0° < x < 180°$. $x = \frac{10}{3}, \frac{190}{3}, \frac{370}{3}$

(e) $2\cos(2x - 70) = 1.2$ for $0° < x < 360°$. $x = 8.44, 61.6, 188, 242$

(f) $\cos(2x + 40) = \frac{1}{2}$ in range $0 < x < 360$. $x = 10, 130, 190, 310$

(g) $\sin(4x - 40) = 0.6$ for $0° < x < 180°$. $x = 19.2, 45.8, 109, 136$

(h) $\sin^2(2x - 30) - 1 = 0$ for $-180° < x < 180°$. $x = -120, -30, 60, 150$

8. In triangle ABC, $AB = 7$, $AC = 11$ and $B\hat{A}C = 100°$.

(a) Find length BC. 14.0

(b) Find angle $A\hat{B}C$. 50.6

9. In triangle XYZ, $XY = 11$, $YZ = 6$ and $Y\hat{X}Z = 20°$.

(a) Find angle $X\hat{Y}Z$. 18.8 or 121

(b) Find the area of the triangle. 10.7 or 28.2

10. Prove the following results

(a) $\dfrac{1 - 2\sin^2\theta}{\cos\theta + \sin\theta} \equiv \cos\theta - \sin\theta$.

11. (a) Find the maximum value of $2 + \sin x$. ☐

(b) Find the smallest positive value of x for which this occurs. ☐

12. (a) Find the minimum value of $5 + 2\cos(x - 40)$. ☐

(b) Find the smallest positive value of x for which this occurs. □

13. (a) Find the maximum value of $2 - 7\sin(2x - 10)$. □

(b) Find the smallest positive value of x for which this occurs. □

14. (a) Find the minimum value of $\dfrac{5}{7 + 3\sin(x - 30)}$. □

(b) Find the smallest positive value of x for which this occurs. □